Introduction to Class AM/AW Mathematics

1 This schedule is the result of a rigorous and detailed analysis of the vocabulary of mathematics, using the techniques of facet analysis. As such, it represents a radical revision and expansion of Class AM/AX in the first edition of the Bibliographic Classification (BC1). The general reasons for making the revision so radical a one are given in the *Introduction and Auxiliary schedules* (Butterworths, 1977). The particular changes in this class are considered in Section 15 below.

2 The Outline on page lxxxi is designed to give a clear view of the basic structure. If it is remembered that the schedule is an inverted one (see Section 8), the outline will be seen to show not only the general sequence of categories and their classes but also the basic operational rule in applying the classification. This is the rule that compound classes (those reflecting the intersection of two or more simpler classes) are usually located under the class appearing *later* (lower down) in the schedule. For example, a work on decomposition of Lie algebras (ATC 8GM) goes under Lie algebras (ATC) and not Decomposition (AM8 GM); Representation of Lie groups (ASJ 9S) is located under Lie groups (ASJ) and not under Representations (AM9 S); Spectral theory of partial differential equations (AWG EN3 A) is located under Partial differential equations (AWG) and not under Spectral theory (AQN 3A).

3 Scope of Class AM/AX and its place in BC2

3.1 The relations of mathematics to logic, to philosophy and to the abstract and empirical sciences have already been touched on in the Introduction to Philosophy and Logic.[1]

In the Introduction to the Bibliographic Classification[2] Bliss described mathematics as

> . . . a general method rather than a branch of science or of philosophy. It has been defined as a 'science of relations', but it is not a science in the sense of knowledge of objective realities; it is rather a method of treating

[1] Class A/AL: Philosophy and Logic / Kenneth Bell and J. Mills. Bowker-Saur, 1991. §3.44.
[2] Volumes I–II, 1952, p.77.

> their relations abstractly, and more especially their numerical and spatial relations. Thus it is a method of dealing with objective realities in these relations.

A more recent definition — but there are many of these — is that mathematics is the study of the logical consequences of sets of axioms or postulates. This is perhaps too broad a definition; but it does emphasise the distinction between mathematics and the empirical sciences which Bliss was making. Just what set of axioms is its province is a big question and it is in respect of this question that Jagjit Singh[3] says

> ... some people propose to cut the Gordian knot by claiming that 'pure' mathematics is merely concerned with working out the consequences of stated axioms with no reference whatever to whether there is anything in the real world that satisfies these axioms.

Certainly modern mathematics tends to become increasingly abstract:

> ... Each major concept embraces not one but many diverse objects, all having some common property. An abstract theory develops the consequences of this property, which may then be applied to any of the diverse objects. (Ian Stewart. Concepts of modern mathematics. Penguin, 1981. p.1.)

3.2 The location of Mathematics as the first of the scientific (as opposed to humanistic) subjects also fits into the overall pattern of increasing concreteness (or decreasing abstraction) — a principle of filing order associated with Ranganathan in particular, but also reflected in Bliss's gradation in speciality. So relations (mathematics) precedes energy (physics) which precedes matter (chemistry) which precedes material complexes (earth sciences) which precede living organisms (biology), and so on.

3.3 It is worth noting here the close relations of logic and mathematics with classification itself:

> ... Yet at the bottom the classifier and the mathematician are doing the same thing, finding and manipulating patterns.

(C.W. Kilmister, in Foreword to J.L. Jolley's The fabric of knowledge: a study of the relations between ideas. Duckworth, 1973.)

The patterns Dr. Kilmister was considering were those advanced by J.L. Jolley, in which logical and mathematical concepts were treated as the preliminary and abstract category prefacing (and thenceforth pervading) the whole field of knowledge as articulated by the theory of integrative levels. It is true, as Dr. Kilmister says, that they

> ... almost as a by-product revealed a scheme for classifying the concepts of mathematics and using them as a means of ordering the other sciences.

[3] Mathematical ideas: their nature and use. Hutchinson, 1959. p.1.

Unfortunately, it was not found feasible to base the BC2 classification of mathematics — or of logic — on Jolley's templet, since the structure of BC2 classes is already well established and we wished to see it manifested in mathematics and logic as well as in all the other classes. But the overall order of BC2 reflects to a large degree an order of integrative levels (of which Bliss's gradation in speciality may be seen as a special version to meet bibliographic indexing ends) and the comprehensively faceted structure meets many of Jolley's criteria.

3.4 Class AM/AW deals only with pure mathematics. Consistently with Bliss's own view of the relations of pure mathematics to applied mathematics, BC2's general principle of Action-Agent favours the subordination of applied mathematics to the subject to which it is applied, e.g. mathematical physics to physics; mathematics of astronomy to astronomy. There is in any event no particular area of mathematical knowledge that can be defined as "Applied mathematics"; each field of interest makes use of what is applicable to its own subject matter. But BC2 provides, of course, an alternative for librarians wishing to keep with mathematics all the literature on applied mathematics too — see Section 9.

3.5 Statistics holds a special position here. It is preeminently an application of probability, a recognised branch of modern mathematics, aimed at producing models of events which are of practical utility. It is therefore considered as a separate class and given its own Introduction — see p. lxv.

4 Structure of Class AM/AW Mathematics

4.1 All classes in BC2 are designed consistently according to a basic pattern which reflects the six fundamental features of a modern documentary classification. In the design operation, these six features are taken in an invariant order in which each step depends on the preceding ones having first been decided. The steps are, in order: (1) organizing the terms into broad facets; (2) organizing the terms in each facet into specific arrays; (3) deciding citation order (between facets and between arrays); (4) deciding filing order (of facets, of arrays); (5) adding notation; (6) adding an A/Z index. These six features are now considered in some detail in Sections 5/11.

5 Facet structure of Class AM/AW

5.1 The main feature of the schedule is a strict adherence to the principles of facet analysis. A facet consists of all the classes produced when the vocabulary is divided by one broad principle of division. So the terms making up the vocabulary of mathematics are initially organized into ('divided into') broad facets. Terms representing concepts which stand in the same broad relationship to the containing class are found in the same facet; e.g. all terms representing the notion of mathematical structures and systems are brought together in one Systems facet; all terms representing the notion of relationships between mathematical concepts are brought together in one

Relations facet; all terms representing the notion of processes within systems are brought together in one Processes facet, and so on.

5.2 Facets in Mathematics AM/AW

The facets identified in this analysis are summarized below; their scope and relations are considered in more detail under citation order — Section 7.2.

[1] Branches and systems of mathematics — algebra, geometry, analysis…

[2] Parts, elements, entities within systems — factors, solutions, boundaries, series… This facet includes Subsystems; unlike the other parts, these are not independently enumerated. They occur only as parts within a particular class.

[3] Properties — spatial, structural, degree, dimension…

[4] Relations — functions, equations, transformations…

[5] Processes — approximation, continuation, decomposition…

[6] Operations — classification, addition, integration…

[7] Methodologies — metamathematics and logic, elementary methods, algebraic methods…

[8] Forms of mathematical presentation — theory, axioms, proofs, models…

[9] Instruments, agents — machines, computers…

[10] Common subdivisions — history, bibliographic form…

6 Arrays within facets

6.1 Most facets contain terms which reflect more than one specific principle of division. For example, mathematical properties may be divided by a number of different principles such as dimension (linear, plane, spatial…), degree (quadratic, cubic…), structure (symmetric, ordered, periodic…), performance properties (invariant, solvable, associative…). Within the Relations facet, different forms of relationship are also manifested: status relations (similarity, equivalence, inequality, reverse…), relations of location (incidence, embedding, immersion…), relations of association (automorphism, endomorphism, isotopy…).

6.2 The terms resulting from division by one specific principle form an array ('subfacet'). A given facet may be made up of a number of different arrays. In many cases, the principle governing an array is given in italics; e.g. in the Properties facet at AND K is the array '*By number of variables*', giving the classes With one variable, Binary, etc.

6.3 Strictly speaking, terms in an array are mutually exclusive and therefore no compounds are possible within it; there is no class, for example, for finite infinity. So the crucial problem of citation order does not arise within arrays — only between them.

6.4 However, it should be noted that in the case of Mathematics, because of the high level of abstraction — both of the terminology and of the relations between terms — the application of very strict principles of division would often result in extremely brief arrays — often simple dichotomies of x and non-x. So a number of such arrays are collocated by broader concepts of association, and in practice there may be compounding within such clusters of arrays; e.g. in the case of structural properties, a given mathematical structure may be both periodic and continuous, although both concepts appear in the same 'array'.

6.5 Order in array refers to the filing order of individual classes within an array; see Section 8.4.

7 Citation order (combination order)

7.1 This refers to the order in which the elements of a compound class — one consisting of more than one element, whether derived from different facets or from different arrays — are combined ('cited') in a heading. Note that the string of terms making up a heading is called a chain — an important concept in faceted classification. Each term in a chain represents a narrowing of the class defined by the preceding term or terms.

7.11 For example, the subject of a document on congruence relations and the ideals of lattices could be represented by the chain

Congruence relations — Ideals — Lattices

However, it could be represented equally meaningfully by five other chains — the total number being $3 \times 2 \times 1$ different combinations, that is, $n!$ where n is the number of elements. Another would be, e.g.,

Ideals — Lattices — Congruence relations

Citation order decides which of the various possibilities will be followed in the classification.

7.12 Combination order reflects the order of application of the principles of division governing the classification of the containing class — in this case, the vocabulary of mathematics. It determines which concepts are subordinated to which. For example, the first chain above implies that ring structures (e.g. ideals) and algebras (e.g. lattices) would be subordinated to (made 'divisions of') the mathematical relationship (here, congruence). Literature on ideals and on lattices would be scattered under the various relationships they manifested. Obviously, the second chain in the example above seems a more helpful one; but keeping the literature on ideals together would be at the expense of scattering that on relationships like congruence.

7.13 Theoretically, there may be no agreement as to which is the best — let alone the 'correct' — citation order. But the demands of a linear order (of entries in a catalogue or bibliography, of documents on a shelf, etc.) require a consistent order to be observed. The result of applying a consistent citation order is that the scattering of some subjects because of their subordination to others (a major and inevitable feature of bibliographic classification) is strictly controlled and the location of quite complex classes (reflecting the intersection of several facets or arrays) is always predictable. Citation order is the most important feature of a classification system. But clear and consistent rules for it can only be expressed in terms of the facets and arrays involved — hence the prior need to organize terms into facets and arrays.

7.14 Even if the order of the file itself were relatively random as to the subjects of documents, as in an accession-order computer file, say, or in an alphabetically arranged file (where the vagaries of the natural language and the unpredictability of the syntactic forms taken by the different words of a subject description make it a relatively random order), a system of connective references is needed for adequate retrieval. A comprehensive picture of the relations to be accounted for can only be established by a faceted organization of the vocabulary and the application of a controlled citation order.

7.2 Citation order between facets

7.21 In all its classes BC2 seeks to observe as far as possible the 'standard' citation order. For each subject, BC2 seeks to determine the primary (the first-cited) facet by looking for the overall systems in the subject which embody the parts, processes and properties, etc. peculiar to the subject, on the principle that the whole integrates its parts and gives them meaningful relations.

Applying this principle to mathematics raises a number of problems, reflecting the high degree of abstraction in the subject and its deliberate aloofness from considerations of the 'real' world or human objectives within that world. Although its containing 'systems' in the above sense are not too difficult to discern, such concepts must be treated with some caution in mathematics. It is necessary to remember, for example, such peculiarities as the fact that there are as many even numbers (say) in the set of reals as there are integers, despite the fact that even numbers would appear to be a subset of the integers, and therefore 'contained within' them. Nevertheless it is possible to define the different branches of mathematics in terms of objectives. We could say, for example, that abstract algebra seeks to establish mathematical structures which display analogues of the arithmetical operations in which it uses symbols to represent the numbers or variables. Or analysis could be said to investigate limiting processes whilst geometry seeks — at least, in Euclidean geometry — to establish idealizations of perceived space and to characterize them algebraically or axiomatically. It was concluded, therefore, that the primary facet in mathematics consists of the well-established branches or systems, such as those in the above examples.

7.22 With the primary facet established, the rest of the standard citation order usually falls into place fairly readily. With the possible exception of the Relations facet this proved to be the case in mathematics. Any given system may display types ('species') of itself (e.g. rings may be compact, or perfect, or associative...), and parts or elements (e.g. boundary, products, residues, extrema, spectra...). Its structures will display various properties (e.g. binary, finite, ordered...), and when taken in combination they exhibit relations between each other (e.g. similarity, reflection, immersion). One may further distinguish processes (e.g. growth, variation, decomposition) occurring within systems, and operations (addition, factorisation, integration...) may be performed upon them. All these features will be examined with the aid of particular methodologies (e.g. classical methods, set theory, mathematical logic, algebraic methods) and the result of such examination will appear in various forms of mathematical presentation (e.g. axioms, theorems, proofs...) whilst all the operations, methods and results may utilize instrumental aids (e.g. computers...). Finally, the literature on all these activities will reflect a number of common subdivisions (e.g. history, professional organizations, bibliographical forms...) found in all subjects.

7.23 The citation order of facets is therefore the order given in the listing of facets in Section 5.2 above. The mathematical branch is the primary (first-cited) facet; parts, elements, etc. are the secondary facet, and so on. This means that a work on distribution in number fields, which reflects two different facets (number fields from the Systems facet and distribution from the Processes facet) will be classed as Number fields — Distribution, and not as Distribution — Number fields.

7.24 Retroactive citation order

7.241 For reasons explained in Section 8 (Filing order) the primary (first-cited) facet is located last in the schedule, the secondary (second-cited) facet files next to last, and so on. So the facet filing first in mathematics (AM2 Common subdivisions) is actually cited last when 'building' classes, e.g. Calculus — History, not History — Calculus.

7.242 It follows from this that the 'building' of compound classes will normally be done by taking a class from a later facet (e.g. Systems facet) and adding to it a class from an earlier one (e.g. Processes facet) — as in the example of Number fields — Distribution above (Section 7.23). This procedure of working backwards through the schedule is usually called 'retroactive synthesis', reflecting 'retroactive citation order'.

7.25 Modifications of retroactive citation order

7.251 As explained in the Introduction to BC2 — Sections 7.331 and 5.734.6 — a number of basic indexing rules are always observed and these sometimes lead to the modification of the usual 'retroactive' rule. Because of the complexity of the relations in the mathematical literature, some of these rules are unusually prominent in this class.

7.252 One in particular affects the order of the classes profoundly. This is the distinction between qualification and specification — a very important one in bibliographical classification. A class is ‘qualified’ when it has as a ‘subclass’ a concept reflecting any relationship other than that of a type of that class e.g. Manifolds — Deformation. This reflects an intersection of two sets: ‘Information on manifolds’ and ‘Information on deformation’. Deformation is a relation arising from an operation on manifolds — it is not a kind of manifold. A class is ‘specified’ when it has as a subclass a concept defining a type (or species — hence the term ‘specifier’) of that class, e.g. Manifolds — Complex. This can also be constructed as an intersection of sets (in classification terms); but the resulting subclass is quite different from its qualification by ‘complex’ as a property, which would represent the class Complex properties of manifolds.

7.253 Terms from any facet may appear as specifiers (species-makers). For example, types of topological spaces defined by an element to give spaces with operators; types of differential geometry defined by a relation to give projective differential geometry; sub-manifolds defined by a property to give non-regular sub-manifolds; semi-groups defined by an operation to give multiplicative semi-groups. A prominent example of the above is the use of the Branches or Systems facet to specify types of methods applied to a particular topic — e.g. topological methods in analysis.

7.254 The practical result of this is that we now get chains in which some terms are followed by terms which file later in the schedule than themselves — i.e., the strict ‘retroactive’ rule appears to be broken. For example, a document on the coordinate density of sets of vectors would get a purely retroactive chain:

> Sets (ARB) — Vectors (AQH) — Coordinates (AQCK) — Density (AN8T)

because all the terms stand in their basic facet relations (Sets as a system, Vectors as an entity, Coordinates as an element, Density as a property).

But a document on inverse elements in symmetric semigroups gets a chain:

> Semigroups (ARY) — Symmetric (ANOJ) — Elements (APA) —
> Inverse (AM9P)

Here, the concept Symmetry (at ANOJ) is cited before the concept Element (at APA), although the latter obviously files later in the schedule. The reason is that symmetry is not acting in its basic facet role as a property (which would give symmetric properties of semigroups) but as a specifier of a type of semigroup — i.e. a symmetric semigroup.

7.255 It should be noted that in the standard citation order Types of anything are always cited before any other facet; they are analogous to the ‘containing system’ which defines the primary facet.

7.256 Another situation in which a facet may be cited before one which files after it in the schedule occurs with the Properties facet. Although this facet files after the Operations, Processes and Relations facets because it applies primarily to the structures in the Parts and Systems facets (e.g. Lattices — Distributivity) it may also be needed to qualify terms from the preceding facets.

7.26 Because of the unusual frequency with which the classifier needs to cite a term outside its normal citation order a special Auxiliary Schedule AM1 is provided to effect this with maximum simplicity and notational clarity. This is described in Section 10.62/10.63 below.

7.3 Citation order within facets (between arrays)

7.31 There are no general principle available as yet for deciding citation order between arrays. Decisions are largely empirical, being based on considerations of where any given compound formed from two concepts within the same facet would most helpfully go. To a limited extent, however, the principles operating in the standard citation order continue to operate within as well as between facets. For example, within the large Properties facet, those derived from other facets are cited in the same order as that used between the originating facets. Also, for those other arrays of properties, the principle of decreasing concreteness — advanced by Ranganathan, the 'inventor' of faceted classification — is observed as far as possible; so more abstract properties are cited after more concrete ones, such as structural and spatial properties.

7.32 The number of different arrays is so large that it is quite out of the question to attempt to list them in citation order as has been done for the main facets (in Section 5.2). However, this citation order is indicated clearly by the inverted filing order (explained in Section 8 below): an array filing later — further down in the schedule — is cited before one filing earlier. A demonstration of this is given in Section 8.31 below; there, the class Finite Abelian groups is cited as Groups — Abelian — Finite, reflecting the fact that the specifying property Finite files before (and therefore cites after) the specifying property (Abelian).

7.4 The overall citation order in mathematics is therefore that given in Section 5.3 above (citation order between facets) amplified within each facet by the order implied by the 'inverted' sequence of arrays within each facet. As mentioned in Section 7.24 above, this citation order of concepts is the reverse of their filing order; this latter will now be explained.

8 Filing order

8.1 This is the order in which the individual classes, simple or compound, file one after the other in the schedule, on the shelves, in catalogues and bibliographies, etc. It has two separate components:

8.11 the order in which the facets, regarded as blocks of terms, file (and within them their arrays, similarly regarded as blocks of terms); this is explained in Section 8.3 below;

8.12 the order of individual terms within each facet; this is explained in Section 8.4 below.

8.2 The distinction between citation order and filing order is a notorious stumbling block for students of modern bibiliographic classification. It is perhaps most easily explained by comparing it with the situation in the ordinary telephone directory. This is usually thought of as being in alphabetical order; but this describes only the filing order, which is only half the story. Equally crucial, although rarely considered, is the order of the constituent elements in each entry. This is basically Surname — Forename or initials, although other components are brought into play (e.g. designations like Dr., Sir) when a large block of names with the same surname and forename needs to be further differentiated.

8.21 Although the problem of citation order when headings are simply the names of persons is quite simple when compared with that of headings representing the names of subjects, the same effects may be observed; e.g. it determines that all the entries under, say, Smith, or Jones or Robinson are kept together, at the expense of scattering those for Tom or Dick or Harry.

8.3 Filing order of facets and arrays (inverted filing order)

8.31 This is determined by the desirability of maintaining a consistent order of general before special. This requires an arrangement known as an inverted filing order. It is best explained by a simple demonstration:

	Relations
AM9 S	Representations
	Branches, Systems
ASA	Groups
ASA 9S	Representation of groups
ASF	Abelian groups
ASF 9S	Representation of Abelian groups
ASF NJ	Finite Abelian groups
ASF NJ 9S	Representation of finite Abelian groups

8.32 It is clear from the subordination of Representation to Groups (and to types of groups) that the citation order is: Branch/System — Relation. But in the filing order, the Relations facet files before the Branches/Systems facet.

8.33 If the filing order were the same as citation order and the Relations facet filed after the Branches/Systems facet, the general class Representations would file after its subclasses (Group representation, etc.). This offends a very widely held expectation by searchers in a systematically arranged file; this is the assumption that the general precedes the special; e.g. nobody expects the general books on British history to file after those on medieval Britain, or 19th century Britain.

8.34 It should be mentioned, perhaps, that a bibliographic class Representation of groups is a subclass of both Representation and of Groups — as symbolization by Venn circles would clearly show. It is only citation order which determines which 'parent' should constitute the containing class and which the subclass.

8.35 In the same way, because terms from different arrays may intersect to form compounds, the filing order of arrays within each facet must also be inverted:

	Properties
	By level of finiteness
ANJ	Finite
	By conformity to fundamental laws
AO9	Commutative, Abelian
	Branches, Systems
ASA	Groups
	Specified by property of finiteness
ASD	Finite groups
	Specified by property of fundamental law observed
ASF	Abelian groups
ASF NJ	Finite Abelian groups

The citation order between the two arrays of Properties is clearly

(By fundamental law) — (By finiteness)

as is seen when they act as specifiers of groups above. If the filing order of arrays were the same as their citation order, the class Commutative would file before the class Finite; consequently the general class Finite groups would file after its subclass Finite Abelian groups.

8.4 Filing order in array

8.41 The classes in an array are mutually exclusive and cannot normally be compounded; so the filing order within an array cannot be determined by citation order. A number of well-known orders in array are used in BC2 where appropriate — e.g., chronological order of periods in history, geographical order of places, etc. These are rarely of any significance in mathematics. Orders of complexity, degree, etc. are used when appropriate — e.g. in a number of arrays of properties in AN. Otherwise, orders in array are pragmatic and of minor importance.

9 Alternative arrangements

9.1 A prominent feature of BC2 is its provision for alternative arrangements in situations where a strong demand for one exists. In such cases, notation is designed to allow for the alternative to be adopted.

9.2 The need for alternatives in mathematics and in statistics and probability proved to be much less than in the case of many other subjects. But in both classes it was clearly helpful to provide for the collocation with the class of all its applications. This has been done; Applied mathematics may go at AWY and Applied statistics at AXY. In each case it would become the primary facet if used. In mathematics, the only notable alternative otherwise is for algebraic geometry, which may be subordinated to geometry at AUI TG rather than algebraic systems at ATG — where it is preferred.

9.3 However, the incidence of specialization in mathematics implies a considerable potential for alternatives whereby specialists in, say recursion theory or combinatorics or topology might wish to bring everything on their subject together.

9.31 Use of such alternatives is facilitated by the provision of Auxiliary Schedule AM1 — see Section 10.63. This schedule was designed, inter alia, to meet the demands arising from the exceptional frequency with which concepts in mathematics change their role in respect of forming compound classes — see Section 7.25. The notational structure of this Auxiliary allows any class to be qualified (or specified) by all others. So although it was not designed for this purpose, a library wishing, say, to keep together everything on a particular branch or sub-branch could do so by using Auxiliary Schedule AM1 to bring in under it all qualifying and specifying concepts, whether these normally file before or after it in the schedules. But to use this facility indiscriminately, abandoning a consistent citation order, is a recipe for disaster.

10 Notation

This is explained in detail in the Introduction to BC2 — especially Section 6.4. Because the notation in the mathematics class is unusual in some ways, the notes given here are somewhat fuller than those usually given for a particular class.

10.1 Notation is a system of classmarks representing the terms — classes — of a classification. Its function is to locate in a mechanical fashion the position of each and every class, simple or compound, in the system. It does this mechanically because its symbols (in BC2, letters and numbers) have a positional or ordinal value already known to the users. The only ordinal value the BC2 user must learn is that a number files before a letter — e.g. AM9 files before AMA.

10.2 Some notations attempt (but always unsuccessfully) to perform a supplementary function. This is to express the hierarchical relations of the classes they represent. For example, in UDC Farming is 631, Farm implements 631.3, Soilworking implements 631.31, Ploughs 631.312, and so on — the classmarks reflect the class containment relation. But it soon breaks down — e.g. Crops are spread over 633/635.

10.21 BC2 notation is purely ordinal; it concentrates entirely on the basic function of indicating order i.e. relative position — e.g.

ATS	Geometry
ATT	Geometric structures
	Associations of structures
AUF B	Fibres

AUF D	Fibre bundles
AUF E	Block bundles

The above chain gives classmarks (ATT, AUEI implied, AUFB, AUFD, AUFE) which do not express the hierarchy to which the concepts belong. On the other hand, the classmarks are shorter than they would otherwise have been; e.g. Block bundles would need a classmark eight characters long if each step of division were expressed in the notation by the addition of another character.

10.3 It should be noted that notation does not determine the order of the classification. The order is determined entirely by the theoretical principles and rules described in Sections 4/9 above. All that notation does is to maintain this order mechanically; it is the servant of the order, never the master.

10.4 The notation is fully faceted and synthetic. Compound classes formed by the coordination (intersection) of two or more 'elementary' classes are given classmarks which are built ('synthesized') from the simpler constituents, taken from their different facets or array. The procedure is described fully in the Introduction to BC2, (Section 7.4) and only the essential points are repeated here.

10.41 The main function of synthesis in notation is to provide maximum 'hospitality' in the classification. Within any subject (e.g. mathematics) the number of potential classes is quite enormous. Every term in every facet is theoretically capable of intersecting with every term in every other facet. Although documents may deal explicitly with a large number of these, this number is still only a fraction of the number the literature might conceivably deal with. The notation must be flexible enough to accommodate all of them (see also Section 10.52 below). It will also, of course, need to be able to accommodate new concepts as they arise. It is assumed that new concepts will always fit into the existing facets, which reflect categories fundamental enough to allow this. But new arrays, which are much more narrowly defined, may sometimes be called for.

10.5 Enumeration of compound classes in the schedule

10.51 Because of the practical impossibility of printing out — 'enumerating' — in the schedule all the compounds possible, a faceted schedule may adopt a rigorous policy of not giving any. For example, the first faceted classification to appear, Ranganathan's Colon Classification, simply listed all the 'elementary' terms in their facets and arrays and left it to the classifier to build the classmarks for all compound classes.

10.52 For a number of reasons, this style of schedule is not observed in BC2. In nearly all its classes it provides a certain number of compound classes, although theoretically the classifier is quite capable of building the classmarks for them. In mathematics, the enumeration of compound classes is in fact much greater than in any other class of BC2. This is explained in detail in Section 10.8 below.

10.53 It may be noted here that the detail enumerated in the schedules AM/AX, whilst considerable, is only a fraction of what the system actually provides by reason of its faceted structure and synthetic notation. When this is taken into account, this class of BC2 is almost certainly the most detailed bibliographic classification of mathematics in existence.

10.6 Classmark building (synthesis) in mathematics

Two main methods are used — retroactive notation and intercalators:

10.61 Retroactive notation is the chief form of synthesis in BC2 as a whole. In this, an earlier classmark (one filing earlier in the schedule) is added directly to a later one, dropping those initial letters or numbers common to both, but without providing an explicit linking symbol. For example, AM5 is Set theory, AM3D is Axioms; so AM5 3D is Axioms of set theory.

10.611 However, it should be noted that in the case of mathematics much of the detail in the BC2 Introduction relating to retroactive notation hardly arises since the synthesis in it is mostly effected via the intercalators given in Auxiliary Schedule AM1. This Schedule includes, however, substantial elements of simple retroactive notation.

10.62 Intercalators (facet indicators) are the second method. Their use means that the addition of a qualifier or specifier is signalled explicitly by using a special symbol (the intercalator or 'facet indicator') to introduce it.

10.621 For example, AM9 I represents Congruence (in the Relations facet). When used in its basic role (as a relation) it is added directly to the classmark of the concept being qualified, but dropping the 'AM' — which is implicit in all the facets AM2/AM9 when these numeral classes are used as qualifiers. So the compound class Congruence of semigroups (where Semigroups is ARY) is ARY 9I. This is an example of simple retroactive notation. From AMA onwards (i.e., all classes filing after the 'numerical' facets AM2/AM9) no class uses 2/9 for enumerated subclasses — the numbers are 'reserved' to allow their direct addition as above.

10.622 But if the same relation Congruence were used as a specifier (see Section 7.25) the subclass it would produce would file in a quite different position from that which it occupies as a relation per se, since Types of anything file after its Relations. To show this, the AM9 in AM9 I is replaced by an intercalator 'L'; so Congruent semigroups would be ARY LI, the 'L' signalling that the specifier it brings is that of Types characterized by relation.

10.623 The frequency with which terms from practically all facets are called on to act as specifiers means that provision must be made for building classmarks 'forward' as well as backward (retroactively). This provision is effected by Auxiliary Schedule AM1.

10.63 Auxiliary Schedule AM1 is designed primarily to allow qualification or specification of any Branch or sub-branch of Mathematics (classes AR/AW) by all preceding facets

(AM/AQ). But the frequent occurrence in the literature of changes in the role of concepts (see Section 7.25) means that concepts from virtually all facets may need similar facilities from time to time. So Auxiliary Schedule AM1 allows any class in mathematics to be qualified or specified by concepts from any other facet where the relationship demands this.

Although Auxiliary Schedule AM1 is self-explanatory, its role in the classification is so crucial that it may be helpful to summarize its operation here. The schedule allows any class in AM3/AWY (AM2 is excluded here since it takes Common Subdivisions, not mathematical concepts) to be divided as follows:

[1] **Qualification by Classes AM2/9** — covering Forms of mathematical presentation, Operations, Processes, and most of the Relations. This is done by the *simple addition of the numbers 2/9* following AM to the class concerned; e.g. to ARR Lattices is added 3KR (from AM3 KR) to give ARR 3KR Word problems in lattices; or 9R is added (from AM9 R) to give ARR 9R Generalization of lattices.

[2] **Qualification by Classes AMA/AQY** — covering some further Relations, all Properties, Elements, Entities. Here, classes are *introduced by intercalators A/E*; ('A' introduces the A/Y following AM; 'B' introduces A/Y following AN, and so on to 'E', which introduces A/Y following AQ. For example, to ASV Fields is added BH (from ANH, the 'B' replacing 'AN') to give ASV BH Galois properties of fields; or DF may be added (from APF, the D replacing 'AP') to give ASV DF Field boundaries.

[3] **Qualification by Subsystems** is introduced by 'F' which is divided exactly like the main systems AR/AW; e.g. to ASX Y Vector spaces is added FSW (the 'SW' coming from ASW Skew fields) to give ASX YFS W Vector spaces over skew fields.

Special subsystems or parts are introduced by 'G' — e.g. ARG Graphs, ARG GE Edges and faces. Note that these are 'enumerated' classes — i.e. they do not reflect compounding of their containing class with other classes in AM/AW.

[4] **Specification by Classes AM3/AM9**. Classes 3/9 (from AM3/AM9) are now *introduced as specifiers by intercalators* H/L; e.g. 'I' replaces AM6, 'J' replaces AM7, and so on. So to ARY Semigroups is added LP (the 'L' replacing the AM9 in AM9 P Inverse) to give Inverse semigroups ARY LP.

[5] **Specification by Classes AMA/AQY**. This is done by *simple addition of the letters* M/Q following the initial 'A'; e.g., to ARY Semigroups is added NN from ANN Regular to give ARY NN Regular semigroups.

[6] **Specification by Systems (Classes AR/AW)**. This is done by *simple addition of the letters* R/W following the initial 'A'; e.g. to ARY Semigroups is added TB from ATB Matrices to give Matrix semigroups ARY TB.

10.64 Note that classmarks which have been modified in AR/AW (see Section 10.7) should not be used in [6] above. For example, under Groups (ASA) a number of prominent types of groups get specially shortened classmarks; so Commutative groups is ASF — instead of ASA O9 which normal specification by a property would give (Commutative being AO9). However, if Commutative groups is itself used as a specifier, the modified (shortened) classmark is not used; e.g. Algebraic groups is ATH — but Commutative algebraic groups would be ATH O9, not ATH SF.

10.641 Another way of putting this is to say that when specifying by AR/AW the only classmarks which are added exactly as they stand are the basic enumerated classes — those not reflecting synthesis with other classes in AM/AW. For example, Manifolds with G-structure is AUG UEG; here, the specifier (G-structures AUE G) is itself a basic, enumerated class. But in Finite group schemes (ATO FSA NJ) the specifier (Finite group ASA NJ) is a synthetic class; although it has been given a special, shorter classmark in the AR/AW schedule (at ASD) this is not used as it stands when used as a specifier since it still represents a synthetic class.

10.65 The above rule does not apply when adding classmarks from AR/AW to –F (Subsystems) or to –6 (from AM6 R/AM6 W) for Mathematical methods. Both these are divided exactly like AR/AW. For example, AVR VJ is Topological groups in algebraic topology; one of its subsystems is Homotopy groups (ASC ML) — which is a modified class in AR/AW. However, this modified version is added directly to –F to denote its role as a subsystem of Topological groups, giving AVR VJF SCM L. A more detailed demonstration of this point will be found under the notes on translation in Section 13.23.

10.66 The fact that numbers 3/9 and letters A/W may be added to any class as intercalators means that only X is left under any class for the enumeration of further types — i.e. ones special to that class (Y is reserved for the option 'Applications'). This is no problem in the purely ordinal notation of BC2; in many cases such special subdivisions of a class are accommodated in the next letter or letters (or, more rarely, next number or numbers). For example, the various types of Transformations (AM9 5) are accommodated at AM9 6/AM9 C; a special type of Fields (ASV) is accommodated at the next letter ASW Skew fields.

10.7 Modification of normal synthesis

10.71 By normal synthesis in the mathematics class is meant the addition of qualifiers and specifiers using the full range of intercalators 3/9, A/W from Auxiliary Schedule AM1 as described above in 10.6.

10.72 Adding together a number of component parts to represent a highly specific compound class inevitably lengthens the classmark for such a class. In the division of the Branches facet, such compound classes are often substantial subjects in their own right and as such are themselves extensively divided. Using normal synthesis would mean adding these further qualifications and/or specifications to a classmark already lengthened by synthesis. So, in a number of such cases, normal synthesis is interrupted by assigning a shorter classmark to the class to be divided and resuming the normal synthesis at a later point. The basic procedure for this is the same in all cases and is best explained by an example (in which a shorter classmark is given to a prominent class, Abelian varieties):

ATL	Varieties
	Types
ATL MY	Rational varieties
ATL P2	*By named mathematician*
	* For Abelian varieties, use ATM.
	* Normal synthesis is interrupted here; it is resumed at ATN.
ATM	Abelian varieties and schemes
	Operations
ATM 7M	Complex multiplication
	. . . ATM is now qualified and specified by
ATN	*Other types of varieties*
	* Normal synthesis is resumed here after interruption at ATL P2.
	* Add to ATN letters P3/W from Auxiliary Schedule AM1.
ATN P3	Picard varieties

10.73 Here, the classmark which the favoured class (Abelian varieties) would get by normal synthesis would be ATL P2 in which 'P2' comes from AP2 in the array 'By named mathematician' in the Properties facet. This is replaced by the shorter classmark ATM and a note is given to this effect. The note also indicates where normal synthesis is resumed and at the latter point (ATN) an 'Add' note shows how the interrupted sequence is picked up without the loss of any class (P3, from AP3, is the next class after AP2).

10.74 In some cases, the interruption is not simply for one class, but for several; e.g.

ASM	Rings
	Types
ASM O7	Associative
	* For Associative rings, use ASN.
	* Normal synthesis is interrupted here; it is resumed at ASR.

ASN	Associative rings
	Methods
ASN 6RU	Homological methods
	Types
ASN PP	Quotient rings
ASO	Non-Associative rings
	* Normal synthesis would give ASM O8 — using O8 from AO8 as specifier.
ASP	Commutative rings
	* Normal synthesis would be ASM O9.
ASQ	Non-commutative rings
	* Normal synthesis would be ASM OA.
ASR	*Other types*
	* Normal synthesis is resumed here after its interruption at ASM O7
	* Add to ASR letters OB/W from Auxiliary Schedule AM1 — e.g.
ASR PRY	Unique factorization rings

Again, normal synthesis resumes with the picking up of the interrupted sequence. The last 'interrupting' class is ASQ, which by normal synthesis would be ASM OA, where the OA comes from AOA Non-commutative, used as a specifier. So the next class to be taken as a specifier, on resuming normal synthesis, is AOB — which is OB in Auxiliary Schedule AM1.

10.75 On a few occasions modification takes a slightly different form. Instead of adding to a shortened classmark all the remaining divisions of Auxiliary Schedule AM1, the divisions of another class only are added. This is seen at AW9 Y and AWA and AWC below:

AW	Analysis
AW8 L	Functions
	Types of functions by property
AW8 NEC	With complex variables
	* Use AW9.
	* Normal synthesis is interrupted here; it is resumed at AWD.
AW9	Functions of a complex variable
AW9 Y	*Types of functions by other variables*
	* Add to AW9 Y letters E/R following ANE.
AW9 YH	Functions of several variables
	* Taken from ANE H.
AWA	*Types by other properties*

	* Add to AWA letters F/XH following AN.
AWA XG	Almost periodic functions
	* Taken from ANX G.
AWA XH	*Harmonic*
	* Taken from ANX H.
	* Use AWB.
AWB	Harmonic functions, potential functions
	Types of harmonic functions by property
AWB NXK	Subharmonic
	* For subharmonic, bi-harmonic and polyharmonic functions, use AWC.
AWC	*Special types*
	* Add to AWC letters K/P following ANX.
AWC K	Subharmonic
	* Taken from ANX K.
AWD	*Other types of functions*
	* Normal synthesis by Auxiliary Schedule AM1 is resumed here after its interruption at AW9.
	* Add to AWD letters NY/W from Auxiliary Schedule AM1.
AWD OD	Convex functions
	* Using –OD in Auxiliary Schedule AM1, which represents class AOD.

10.751 The reason for the modification at AWB is this: the class Harmonic ANX H in the Properties facet has three enumerated (i.e. not synthetic) subclasses ANX K/P. We wish to give each a shorter classmark. So after the interruption at AWA XH, normal synthesis of types of functions is not resumed until AWD.

10.76 It should be noted that the exact order of qualifying and specifying classes got by normal synthesis — using Auxiliary Schedule AM1 — is always maintained, whatever interruptions are made in the interests of shorter notation. Notation is always the servant of order BC2, never the master.

10.77 Also, the full potential for synthesis provided by Auxiliary Schedule AM1 is always maintained. If the reader cares to check the provision above at AW9, AWA, AWB, AWC and AWD they will find that nothing between ANE C (where interruption began) and ANY (which is the first subclass accommodated at AWD NY when normal synthesis is resumed at AWD) is missed.

10.8 Enumeration of synthesized compound classes in the schedules

As noted in Section 10.5, although compound classes are not usually enumerated in a faceted classification, this principle is greatly modified in the mathematics schedule. The reasons for this are as follows.

10.81 A skeleton schedule, consisting only of the ‘pure’ facets, from Forms of mathematical presentation (at AM3) to Branches and systems (at AR/AW) and leaving all synthesis to be done by the indexer as called for, would not convey adequately the detailed structure of the scheme. The fuller structural map of the subject which we have sought to give by enumerating many compound classes should be much more helpful to the indexer when fitting complicated compound classes into it.

10.82 There is a high incidence of classes in which a concept from a given ‘pure’ facet plays a role different from its basic one. This produces so many permutations and variations of the standard citation order that an indication of where such compounds are to be located seems essential to the convenience of the indexer.

10.83 Mathematical terminology is such that it is frequently the case that only enumeration of the full name of the class as it is usually referred to in mathematics conveys clearly and readily the import of the class in question.

10.84 To demonstrate these points, we give below an example of how the schedule not only spells out the hierarchy leading to a particular compound class but also amplifies the bare structure of the hierarchy by giving the generally-used mathematical term where this seems to be called for.

10.841 Before demonstrating this, it seems worth showing just how bare the schedule would appear if it had no enumeration of compounds. All that would appear at ATL Varieties would be:

ATL	Varieties
ATO B	Subvarieties
ATO D	Intersections
ATO F	Schemes
ATP	Cycles and subschemes

Everything else to be found in the schedules at ATL/ATP can be got by synthesis.

10.842 An amplified hierarchy showing a limited number of frequently occurring compound classes but giving only the elementary compound terms within their facets would be:

ATL	Varieties
	Elements
ATL EB	Points
ATL EBM Y	Rational
	Types

ATL P2	*By named mathematician*
	* For Abelian varieties, use ATM
ATM	Abelian
ATM F	Subsystems
ATM FUA	Spaces
ATM FUA NOG	Homogeneous
ATM FUA NOG MTL	Principal

This is certainly clearer than giving only ATL Varieties and leaving the indexer to synthesize all the subclasses — including such precise compounds as Principal homogeneous spaces in Abelian varieties.

10.843 But it is even clearer when this is amplified further, by spelling out at strategic points the compound classes themselves:

ATL	Varieties, algebraic varieties
	Elements
ATL EB	Points
ATL EBM Y	Rational points on algebraic varieties
	Types
ATM	Abelian varieties & schemes
ATM F	*Subsystems*
ATM FUA	Spaces
ATM FUA NOG	Homogeneous spaces of Abelian varieties
ATM FUA NOG MTL	Principal homogeneous spaces

10.844 The above example also demonstrates an additional reason for enumeration, at least of prominent subclasses; this is the interruption of normal synthesis in order to secure shorter classmarks (Section 10.7 above). Both the reasons for and the fact of such interruption is made much clearer by extensive enumeration.

10.9 Notation for multiple entry in the classified catalogue

10.91 This question is considered after Practical classification, in Section 14.

11 Alphabetical subject index

11.1 The function of the A/Z index in a classified indexing system is considered in the Introduction to BC2; Section 6.5 gives general principles and Section 7.5 practical guidance to a library making its own A/Z index to its stock. Below, only the essential points are given, as a matter of convenience. Notes on the efficient use of the printed index which follows the schedules AM/AX will be found on the page preceding it.

11.2 The basic rules for both constructing and using an A/Z index are those of chain indexing. Ideally, an A/Z index consists only of single words, each one followed by the classmark of the subject named. But even if the vocabulary indexed had no compound classes there would be cases in which the same word has several classmarks against it — i.e. the topic indexed appears in several different contexts in the classification. To show the user what these contexts are, such words must be qualified by another term or terms. The basic rule of chain indexing is that all such qualifying terms must be from superordinate classes. An entry term (the one at the front) is never qualified (followed by) a subordinate term — i.e. representing one of its own subclasses in the classification; e.g.

Lie algebras ATC
but not
Lie algebras, Compact ATCNR

The classname Compact Lie algebras will get its own entry under Compact Lie algebras ATC NR. Here, the term 'Compact' is qualified by the superordinate containing class (Lie algebras) which shows what 'Compact' refers to.

11.21 Occasionally, however 'anti-chain' entires will be found in the mathematics index owing to the limitations of the computer program (see next paragraph).

11.3 The A/Z index to class AM/AX has been produced largely by automatic selection of terms from the schedule, using a computer program written to this end — and including, for example, rules for deleting 'anti-chain' entries. Entry terms are qualified only when it is necessary to distinguish the different contexts involved when the same entry term leads to more than one classmark.

11.4 The A/Z index to the mathematics class provides entries for compound classes to a degree not usually contemplated in the index to a faceted classification. The reason for this is the same as for the exceptional number of compound classes enumerated in the schedule (see Section 10.8). This is the high incidence of terms acting in more than one relational role.

11.5 Usually, in the printed index to a faceted schedule, it is assumed that the user (i.e. the indexer or other librarian) is completely familiar with the way in which the classmark for a compound class is built by adding one concept to another. So few compounds are indexed directly; in the mathematics class, e.g., if Points occurs in the Elements facet (at AQB) and Varieties in the Branches facet (at ATL) and the citation order is Branches — Elements then the indexer knows that the Elements concept is added to the Branches concept (and not vice-versa) and that the notational link is made by following the instructions in Auxiliary Schedule AM1. Here, the second component (Points) is added to the first component by replacing the AQ of AQB by 'E'; this gives the compound classmark ATL EB and the indexer knows exactly where it goes. There is no need, therefore, for an index entry:

Points : Varieties ATLEB

11.6 But in view of the complexity of the relations and the ambiguity of the phrasing so often found in the mathematical literature, it was decided to modify the normal practice (designed for maximum economy in A/Z index entries) by providing entries for a number of compounds. In particular, when a concept from a given facet changes its role (usually, to act as a specifier) it seems sensible to treat the resulting compound as a class calling for its own index entry in its own right. So all compounds built from two or more components to represent a Type of anything (i.e., the later component acting as a specifier) are given an entry under the specifier. A simple example is Inverse semi-groups ARY LP. Here, the Branch concept (Semi-groups ARY) is specified by the Relation concept (Inverse AM9 P) to give a particular type of Semi-group. Following Auxiliary Schedule AM1, the AM9 is changed to L to signify the change of role from Relation to Type (specification). The A/Z entry in the direct form is:

Inverse semigroups ARYLP

But no entry would be made for (say)

Inverse problems in partial differential equations AWG9P

since 'Inverse' here appears in its normal role, as a Relation. Similarly, if the class Spaces (a subsystem) in the topology of manifolds (AVO GFU A) is specified by the Relation 'Covering' (AM9 V) it would get an entry:

Covering spaces in topology of manifolds AVOGFUALV

But no entry appears for (say)

Coverings of algebraic curves ATJUU9V

in which Coverings appears in its normal role, as a Relation.

Sometimes, the name of the specified compound does not have a conveniently direct form; e.g. Classical spaces with connections reflects the specification of Classical spaces (AUQ) by the element concept Connections (AQ9) to give the compound classmark AUQ Q9. But the basic principle of chain indexing requires the A/Z index entry to appear under the distinguishing term (here, Connections); so the natural language form of the class name of the compound is permuted by the computer program to give

Connection
  Classical spaces with ~

(where the tilde replaces the term 'Connection').

But again, no entry will appear under

Connections in fibre spaces AUSFCE9

in which Connections appears in its normal role, as an element.

11.7 Another exception to the rule for indexing compounds is when the normal synthesis has been interrupted and the compound receives a classmark other than the one it would have got by simple application of Auxiliary Schedule AM1. Most interruptions are made to get shorter classmarks for Types and would in any case be indexed by the rule in 11.5 above. But sometimes classes in other facets are interrupted — e.g.

Fields in algebraic number theory ARN (would normally be ARLRSFSV)
Geometry of numbers ARJ X (would normally be ARJFTS)
Manifolds in differential topology AVV (would normally be AVUYJRFUG)

11.8 The indexing of interrupted classes does not extend to normally synthesized classmarks following an interrupting class. It is assumed that these can be traced relatively easily by observing the form of schedule presentation described in Section 10.4 above. For example, the class Rings of endomorphisms of Abelian varieties has the classmark ATM FSM DXA D. Having established (from the A/Z index, or from direct consultation of the schedules) that Abelian varieties is ATM, the rest of the classmark is easily constructed by simple application of Auxiliary Schedule AM1. But an exception is made to this rule in the case of a few particularly prominent class; e.g. Differential equations in analysis AWE ME (for which normal synthesis would give AW9 ME) is indexed.

11.81 When interruptions include a special instruction to 'divide like' a particular class (or sequence of classes) the resulting classes are indexed as an extra precaution. For example, at AWE (Other relations in analysis) the special instruction appears: Add to AWE numbers and letters 3/ME following AM9. In this case, A/Z entries are made for the resulting main divisions (Transforms AWE 4; Equations AWE L, etc.) since these now have classmarks differing in construction (albeit slightly) from the normal.

12 Special problems in the classification of mathematics

12.1 A number of special problems have already been considered under particular parts of the explanatory Sections 1/11 — e.g. under citation order, notation, practical classification. Here, we consider some particular problems met with in the design of the schedules but consideration of which in Sections 1/11 would have impeded the purely explanatory nature of those sections.

12.2 A central problem follows from that feature of the classification which marks it off so radically from all other documentary classifications of mathematics: this is the organization of its large vocabulary into a number of broad facets and specific arrays consistent with the structure provided so successfully for every other class in BC2.

12.21 All existing classifications of mathematics plunge straight into an enumeration of branches and disciplines, to which all concepts are subordinated. Where provision is made for a 'generalities' class, this consists largely of classes belonging to the foundations of mathematics (e.g. mathematical logic and sets) and certain branches such as combinatorics which are considered not to be unitary classes but ones servicing many others. A few specific concepts of general application (e.g. approximation and expansion, algorithms, models) may also be found in such a preliminary class.

12.22 BC2, on the other hand, has extracted from the various branches and disciplines a more or less complete vocabulary of mathematical operations, processes, properties, elements, etc. and organized them as rigorously as possible into reasonably well-defined categories.

12.23 The reason for doing this, as in all faceted classifications, is in order to provide the essential basis for a consistent citation order, governed by relatively simple rules. A consistent citation order is the sine qua non of predictable retrieval from a pre-coordinate information system (i.e. the linear files found in catalogues and bibliographies, on shelves, etc.). There is no known way of achieving a consistent pattern of division within all the classes of a subject field other than to combine terms from explicitly identified categories in an explicit order (citation order, combination order). This applies to mathematics just as much as to any other subject.

12.24 The categories identified are listed, and their citation order given, in Section 7.2. The resulting pattern of division of a class (taking into account some complications in citation order which are more prominent in mathematics than in any other class) is summarized in Auxiliary Schedule AM1.

12.3 In most subjects, there is a substantial body of literature relating to all the facets independently and are not necessarily in combination with other facets (especially the primary facet). For example, in Medicine, the Agents (personnel, institutions such as hospitals, medical equipment etc), the Operations (diagnosis, therapy, monitoring, etc.) the Processes (physiological, pathological, etc. together with their dependent agents) and so on all have a considerable literature on them per se and are not necessarily compounded with a particular part or system of the body, or with a particular type of person.

12.31 This is not the case in mathematics. Most of the literature dealing with the Methods, Operations, Processes, etc. considers these in the context of a particular Branch and although most of these concepts (occupying Class AM/AQ) clearly have an independent existence, they are usually thought of more in connection with a Branch than in isolation. For example, functions, operators, series, etc. are usually associated with analysis; spaces and spatial structures are usually associated with geometry or topology. Many of the Properties (particularly of performance and composition) are associated with algebraic structures, as are the associative Relations (e.g. automorphisms, transformations). Most of the Operations are computational and thus arithmetic in nature.

12.32 Nevertheless, although many of these concepts in AM/AQ are unlikely to have much (or in some cases, any) literature considering them per se and in general (i.e., multi-contextually) this is no argument for failing to recognise their independent existence and to provide for their availability as qualifiers or specifiers (see Section 10.23) wherever they are needed. For example, Euclidean space does not appear solely in Geometry; in some cases the very term 'space' may occur without any direct relevance to Geometry (as in Abstract space in functional analysis, where space is defined primarily in terms of sets where a limiting process is defined); series do not appear solely in Analysis. Were such concepts to appear only as subclasses of a particular Branch or disciplines, they could not be considered in a truly general manner and if they are required elsewhere in the classification they could not be extracted without that special context affecting their significance.

12.33 However, there may be cases where the indexer will still prefer the context of a particular Branch as the 'general' class for a concept; e.g. potential theory may be preferred under Harmonic functions (AWB 3A) rather than as a general theory at AM3 CWB — where it is defined by its major application.

12.4 Maintenance of a consistent citation order

12.41 There are two basic assumptions behind a strict rule for combination order.

12.411 The sequence of concepts in their facets and arrays has been worked out so carefully that in most cases the resulting combination order in a given class (got usually by working retroactively) will give it a location which is more helpful than, or least as good as, any other;

12.412 Even if the combination order on some occasions seems less than ideal, the ability to predict exactly where any hypothetical compound subject will go is so crucial in retrieval that this price is worth paying.

12.42 The principle works very well when compounds reflect normal facet relations — e.g. subordinating elements or the parts of a thing to that thing; subordinating operations on things to the thing operated on.

12.43 But as we have seen, an important use of the 'preliminary' facets AM/AQ is to act in the role of specifiers of types of things. For example, out of some 200 concepts in the Properties facet, only 30 or so appear in the schedules in the role of property per se; in the great majority of cases they appear as specifiers. In this situation, when one examines the very large number of arrays from which concepts can be drawn and considers the enormous range of combinations these can generate, the assumption in Section 12.411 implies a very tall order indeed. For example, in types of geometries, projective differential geometry (AUX JR) reflects the fact that the specifier drawn from the Relations facet (Projection AM9 9) is cited before that from the Operations facet (Differentiation AM7 R); Affine differential geometry (AVC JR) reflects the specifier drawn from the Properties facet (Affine AOL) being cited before the same Operation. Conceptually, both these make as much sense subordinated to Projective geometry or Affine geometry as to Differential geometry: but the decision is made by observing basic citation order. But in the case (say) of local differential geometry (AUV NF), where there is no class 'Local geometry' that the compound could go under, the basic citation order (which would cite the Property specifier 'local' before the Operation specifier, is overruled.

12.44 Another example of the occasional overruling of the strict citation order between arrays when used as specifiers is under Rings (ASM). Here, the essential characteristic of division would seem to be the 'fundamental laws' observed (giving Associative, Commutative, etc). So when compounding with other specifying facets or arrays these are cited first, even where the retroactive rule would cite them earlier; e.g.

> Rings — Commutative — Arithmetic (ASP RI); Rings — Associative — Quotient (ASN PP);

here, the subordinated specifiers come from the Systems and Elements facet respectively, whereas the fundamental laws reflect the Properties facet and would normally be cited after Systems or Elements.

12.5 When a branch or system is involved, the different roles must be carefully distinguished. For example, although groups are defined as algebraic structures, class formation in the literature of mathematics is such that the general systems term 'algebra' appears in several differently related intersections with the term 'group'. Used to specify a property, we get Algebraic properties of groups; used to specify a Subsystem, we get Algebra of groups; used to specify a type, we get Algebraic groups. Moreover, the roles of the two branches could be reversed to give Group properties of algebras, Groups of algebras and Group algebras.

12.6 Mathematical terminology and subject analysis

12.61 These two factors in indexing interact to produce a number of problematical situations.

12.62 The verbal form may give a misleading impression of the hierarchical relations; e.g. 'Algebraic geometry' sounds like a type of geometry, whereas its primary concern is the elucidation of algebraic problems involving several variables; the geometry is essentially acting in an instrumental relationship to the algebra. Or, a compound class Birational transform of minimal models suggests that the transform is of the model, whereas the relationship is actually one of the model being used as a demonstration of the transform — so is cited after it.

12.63 The terminology often leaves a key term unstated, being implied by the context; e.g. in the context of analytic spaces as a type of space in topology, the term 'complex space' (AVM) is used when complex analytic space is implied. Similarly, when qualifying non-linear functions by operators (AWR 8X), it is taken as implied that the operators are non-linear. So when clarifying such concepts the implicit concept is not added as a further qualifier or specifier.

12.7 Methods and structures

12.71 It was found necessary to distinguish carefully between two different senses of some Branches — e.g. Algebra. In one, it is viewed as a method or discipline (e.g. Algebraic) and on the other it is viewed as a structure (e.g. algebras). Part of this problem is the decision to subordinate the notion 'Theory' to the more 'concrete' concept when the latter specifies the theory; e.g. Ramification theory of local fields is treated as Local fields — Ramification — Theory. So a class like Algebraic theory of affine geometry might be construed as Affine geometry — (Subsystems, structures) Algebras — Theory. However, this would reflect a failure to distinguish algebra as a method or discipline from algebras as mathematical structures — i.e. collections of mathematical entities sharing particular characteristics, conforming to certain mathematical laws, performing in a particular fashion, and so on. Algebra as a method or technique or 'discipline' on the other hand implies rather the application of a generalized and abstract type of arithmetic. Applied to geometry (for example, in Cartesian geometry) it deals with the properties of geometric figures, curves, etc. by reducing them to a series of algebraic functions and equations. So a parabola, for example, is both the physical form of a curve and a function (given by the equation $y = 2x$) which relates the two coordinates. Traditional geometry uses pragmatic and empirical methods to discover its properties whereas algebra considers the abstract relationships of its elements.

13 Practical classification in mathematics

13.1 The operation of assigning classmarks to documents is considered in detail in the Introduction to BC2 (Section 7). Here we give only the main points before demonstrating these with actual titles.

Three distinct operations are involved.

13.11 Concept analysis

13.111 This means examining the document to decide which concepts provide for a statement of its overall specific subject. This statement (or sentence) should describe what the document is about, using the classifier's own words or words taken from the document. It should not be restricted by the vocabulary of any particular indexing language; CIP (Cataloguing in publication) subject data, in particular, should not be accepted uncritically.

13.112 In formulating this subject statement, it is helpful to ask various questions to ensure that all the essential concepts are incorporated. The questions are best posed in the order of the facet structure of the class. For example, is the document restricted to a particular branch or system of mathematics? Does it deal with a particular part of a system, or property of it? Does a given concept reflect its normal relationship (as a system, a property, a method, etc.) or is it being used as a specifier designating a particular type of another concept? Careful concept analysis is the essential basis of accurate subject classification by any scheme.

13.113 It is helpful to set down the chosen terms, representing the essential subject concepts, in a list or line.

13.114 Some of the problems associated with choosing these terms and the question of just how specific the summarization might be are considered in a separate note at 13.4, after the practical examples. The latter assist a realistic appreciation of the problems considered in Section 13.4.

13.12 Citation order

13.121 This is deciding the order in which the terms decided above should be cited (combined) according to the rules described in Section 7. The terms, when placed in their citation order, constitute a chain — see Section 7.1.

13.13 Translating the chain into notation

13.131 This is mainly a question of applying Auxiliary Schedule AM1 (see Section 10, particularly 10.63). The six basic situations distinguished in 10.63 are demonstrated first.

13.2 Demonstrations of the six basic operations in compounding classmarks

13.21 Qualification by a class taken from AM2/AM9

Subject: Decomposition of Lie algebras

Concept analysis: Decomposition / Algebras / Lie algebras

Citation order: Taking first the class appearing latest in the schedule, the chain is: (System) Lie algebras (ATC) — (Relation) Decomposition (AM8 GM)

Translation: (1) The classmark for the first-cited term is always taken just as it stands in the schedule — here, ATC. (2) All subsequent (added) terms are modified, whether they are qualifying or specifying the preceding term. Just how they are modified is shown in Auxiliary Schedule AM1. (3) Here, there is only one qualifying term — Decomposition. Its classmark (AM8 GM) shows that it comes from the facets in AM3/AM9. (4) When any class in AM2/AM9 is used as a qualifier (i.e. acts in its normal role) the initial AM is dropped and the number added directly to the preceding class. For the example above, this gives ATC 8GM.

13.22 Qualification by a class taken from AMA/AQY

Subject: Cohomology of groups

Concept analysis: Cohomology / Groups

Citation order: Taking first the class appearing last in the schedule, the chain is: (System) Groups (ASA) — (Relation) Cohomology (AMK)

Translation: (1) There is only one qualifier — i.e., cohomology. Its classmark (AMK) shows that it comes from the facets in AMA/AQY. (2) When any class in AMA/AQY is used as a qualifier (i.e. acts in its normal role) a distinctive intercalator ('facet indicator') is needed, to show which facet it comes from. For this purpose the initial AM is replaced by an 'A' when its classmark is added to the preceding one. For cohomology of groups this gives ASA AK. (3) A qualifier from ANA/ANY would replace its AN by a 'B' — and so on (see Auxiliary Schedule AM1 for the complete list).

13.23 Qualification by subsystem

Subject: Galois groups in fields

Concept analysis: Galois / Groups / Fields

Citation order: (1) Although Fields happens to file later than Groups, the citation order is not determined by this fact but by the relationship of System — Subsystem, whereby a subsystem is always subordinated to its containing system. Here, Galois groups are treated as subsystems of fields. (2) So the chain is: (System) Fields (ASV) — (Subsystem) Groups (ASA) — (specified by property) Galois (ANH)

Translation: (1) When a system (from AR/AW) is used to qualify anything as a subsystem, its initial letter 'A' is replaced by the intercalator F. (2) If the system so used is itself qualified or specified by a further concept (as Group here is specified by Galois) the latter is taken exactly as it is scheduled (or would be scheduled) in AR/AW. The extensive use of interruptions in AR/AW means that qualifications and specifications may differ from the constructions got by using Auxiliary Schedule AM1 (see Section 10.641). (3) The property (Galois) here acts as a specifier of a type of group (see Section 13.25 below for the rule).
(4) Reference to the schedules in AR/AW shows that the notation for Groups extends over ASA/ASL and the 'NH' for Galois is added to ASC, not ASA. (5) So the final classmark is ASV FSC NH.

13.24 Specification by a class taken from AM3/AM9

Subject: Additive categories

Concept analysis: Additive / Categories

Citation order: (1) The verbal form indicates that the concept 'additive' is defining a type of category (i.e., acting as a specifier). (2) Take first the concept which is being specified — here, Categories (ASX). In the great majority of cases this will also be the class appearing later in the schedule — but not necessarily so. (3) Take next the specifier. This will always be derived from some term in the classes AM3/AW. In this case, Additive is clearly derived from Addition (AM7 J), in the Operations facet. (4) So the citation order of the chain will be: Categories (ASX) — Additive (derived from Addition AM7 J)

Translation: (1) The specifier is derived from AM7 J, which comes from the facets in AM3/AM9. (2) When a class from AM3/AM9 acts as a specifier, the change in role is indicated by replacing the first three characters (AM and the initial number) by a special letter, taken from Auxiliary Schedule AM1. Classes in AM7 are replaced by 'J'; so the classmark for Additive categories is got by adding to ASX the classmark JJ — giving ASX JJ.

13.25 Specification by a class from AMA/AQY

Subject: Local categories

Concept analysis: Local / Categories

Citation order: (1) The verbal form indicates that the concept Local is defining a type of Category (i.e. acting as a specifier). (2) The concept specified is taken first; here, it is Categories (ASX). (3) The specifier is taken next. Local is clearly derived from Local at ANF in the Properties facet. (4) So the citation order is:
Categories (ASX) — Local (derived from the property ANF)

Translation: (1) The specifier is derived from ANF. This is located in the facets in AMA/AQY. (2) When a class from AMA/AQY acts as a specifier, the change in role is indicated by dropping the initial 'A' and adding the remainder directly to the preceding classmark. So the classmark for Local categories is ASX NF.

13.26 Specification by a class taken from AR/AW

Subject: Group schemes

Concept analysis: Groups / Schemes

Citation order: Schemes (a type of Variety) is being specified by Group — so is cited first. The citation chain is: Schemes (ATO F) — Group (ASA)

Translation: 1. ASA Groups is an enumerated class (not synthesized); so it is added directly, as it stands in AR/AW, to ATO F, dropping the initial 'A'. This gives ATO FSA.

13.27 The six different situations demonstrated above cover virtually all those encountered when building (synthesizing) classmarks in the mathematics class. They also show that synthesis in AM/AW invoves essentially only two operations: either an initial part of the classmark being added is dropped, or it is replaced by one letter (the intercalator).

The six situations will be seen to occur over and over again in the examples below.

13.3 Demonstrations of classification using selected titles

13.31 The following titles are chosen primarily to demonstrate the problems of relations between constituent concepts in compound classes and the accompanying notational problems of synthesis.

13.32 Many of them reflect relatively specific subjects and when the number of constituent elements gets large the classmarks get longer also. It should be remembered that a great deal of the literature, particularly at the book level, does not call for such extensive compounding and the length of classmark found in these demonstration titles should not be taken as typical.

13.33 The titles are arranged by their final classmark — i.e. in the order in which they would file in a classified catalogue or bibliography.

13.34 Two versions of the classmark are given in the translation step; the first breaks it into its constituent elements, in order to show more clearly how the bits are added together. The second, put in parentheses, breaks it into regular blocks of three characters; this is easier to follow when scanning a classified file and is the recommended form for stating BC2 classmarks on documents, catalogue entries, etc. — see Section 13.8.

13.35 When taking the final (translation) step, remember that virtually all synthesis in mathematics is determined by the six situations demonstrated in Section 13.21/13.26. These are summarized below for convenience; the terms 'drop' and 'replace' on the right refer to the way in which the qualifying or specifying classmark is modified when it is added to the preceding classmark.

1. Qualification by a concept from AM2/AM9: drop AM
2. Qualification by a concept from AMA/AQY: replace first 2 letters by intercalator from A/E
3. Qualification by a subsystem from AR/AW: replace A by intercalator F
4. Specification by a concept from AM3/AM9: replace first 3 characters by intercalator from H/L
5. Specification by a concept from AMA/AQY: drop A
6. Specification by a concept from AR/AW: drop A

[**1**] **Title:** *Conditions for equality of decomposable symmetric tensors*

Concept analysis: Equality / Decomposition / Symmetric / Tensors

Chain: Tensors (AQI) — Symmetric (ANOJ) — Decomposition (AM8GM) — Equality (AM9L)

Classmark: AQI NOJ K GM 9L (AQI NOJ KGM 9L)

Comments:

1. Use of Property (Symmetric) as specifier: drop initial A.
2. Use of Process (Decomposition) as specifier: replace AM8 by K.
3. Use of Relation (Equality) in normal role: drop initial AM.

[**2**] **Title:** *Some remarks concerning irregularities of distribution of sequences of integers in arithmetic progressions*

Concept analysis: Distribution / Sequences / Integers / Arithmetic / Progressions

Chain: Arithmetic (ARI) — Progressions (AQP) — (Subsystems) Integers (ARKF) — Sequences (AQP) — Distribution (AM8A)

Classmark: ARI E QRI F RKF EP 8A (ARI EQR IFR KFE P8A)

Comments:

1. Note at AQP says AQQ is used for types of sequences.
2. At ARI EP (Progressions in arithmetic), arithmetic progressions is enumerated as a compound class at ARI ERI.
3. F from Auxiliary Schedule AM1 is used to introduce Integers as a subsystem, dropping initial A.
4. Entity (AQP) and Process (AM8 A) in normal role, replacing AQ by E and dropping initial AM from AM8 A.

[**3**] **Title:** *Coordinate density of sets of vectors*

Concept analysis: Co-ordinates / Density / Sets / Vectors

Chain: Sets (ARB) — Vectors (AQH) — Co-ordinates (AQCK) — Density (AN8T)

Classmark: ARC QH E CK B 8T (ARC QHE CKB 8T)

Comments:

1. The facet (Types of sets) begins at ARC — so this begins the classmark (see schedule).

2. Use of Entity (Vectors) as specifier: drop initial A.
3. Use of Element (Co-ordinates) in normal role: replace AQ by E.
4. Use of Property (Density) in normal role: replace AN by B.

[4] **Title:** *Classes of graphs that are critical with respect to coverings*

Concept analysis: Graphs / Critical / Coverings

Chain: Combinatorics (ARD) — Graphs (ARG) — Critical points (AQCR) — Covering (AM9V)

Classmark: ARG E CR 9V (ARG ECR 9V)

Comments:

1. ARG implies its containing class ARD — so the latter does not appear in the final classmark.
2. Use of Element (AQC R) in normal role; 'critical' implies the critical points of the graph. Replace AQ by intercalator E.
3. Use of Relation (Covering) in normal role: drop AM.

[5] **Title:** *Ultra-metric calculi : introduction to P-adic analysis*

Concept analysis: Ultra-metric calculi / P-adic analysis / Number theory

Chain: Number theory (ARJ) — P-adic numbers (ARKT) — Analytic methods and Calculus (AM6W) — (Special forms — Ultra-metric) (AM745)

Classmark: ARKT 745 (ARK T74 5)

Comments:

1. ARKT implies its containing class ARJ — so latter does not appear.
2. AM6 X/AM7 4 are reserved for special methods; they have been filled out under Analysis (AW) for types of calculi. This expansion is used here; so AM7 45 implies AM6 W (which it immediately follows).
3. Use of Methods (Calculus) in normal role: drop AM.
4. AM7 45/AM7 49 are reserved for special methods in Analysis (see note in schedules at AM6 W).

[**6**] **Title:** *On the connection between congruence relations and the neutral ideals of lattices*

Concept analysis: Congruence / Neutral / Ideals / Lattices

Chain: Lattices (ARR) — Ideals (ASS) — Neutral (AMQD) — Congruence (AM9I)

Classmark: ARR F SS MQD 9I (ARR FSS MQD 9I)

Comments:

1. Ideals is a subsystem here (derived from AR/AW): replace A by F.
2. Use of Property (Neutral) as specifier: drop initial A.
3. Use of Relation (Congruence) in normal role: drop AM.

[**7**] **Title:** *Numerical solutions for non-linear equations*

Concept analysis: Algebra / Numerical / Solutions / Non-linear / Equations

Chain: Elementary algebra (ART) — Equations (AM9L) — Non-linear (ANB) — Solutions (APG) — Numerical methods (AM6RI)

Classmark: ART 9L NB D G 6RI (ART 9LN BDG 6RI)

Comments:

1. Use of Property (Non-linear) as specifier: drop initial A.
2. Use of Element (Solution) in normal role: replace AP by intercalator D.
3. Use of Method (A6RI Numerical) in normal role: drop AM.

[**8**] **Title:** *Inverse elements in a finite symmetric semigroup*

Concept analysis: Inverse / Elements / Finite / Symmetric / Semigroups

Chain: Semigroups (ARY) — Symmetric (ANOJ) — Finite (ANJ) — Elements (APA) — Inverse (AM9P)

Classmark: ARY NOJ NJ D A L P (ARY NOJ NJD ALP)

Comments:

1. Use of Properties (Symmetric, Finite) as specifiers: drop initial A in each case.
2. Use of Element (Elements) in normal role: replace AP by D.
3. Use of Relation (Inverse) as specifier: replace AM9 by L.

[**9**] **Title:** *Periodic products of groups*

Concept analysis: Periodic / Products / Groups / Algebra

Chain: Algebraic structure (ARX) — Groups (ASA) — Product (APM) — Periodic (ANXF)

Classmark: ASA D M NXF (ASA DMN XF)

Comments:

1. ASA implies its containing class ARX — so latter does not appear.
2. Element (Product) used in normal role: replace AP by intercalator D.
3. Property (Periodic) used as specifier: drop initial A.

[**10**] **Title:** *Abelian group extensions and the axiom of constructability*

Concept analysis: Abelian / Groups / Extensions / Axioms / Constructability

Chain: Groups (ASA) — Abelian groups (ASF) — Extensions (AM9E) — Constructability (AMN7D) — Axioms (AM3D)

Classmark: ASF 9E A N7D 3D (ASF 9EA N7D 3D)

Comments:

1. ASF implies its containing class ASA — so latter does not appear.
2. Use of Relation (Extensions) in its normal role: drop AM.
3. Use of Property derived from another facet; Constructability is derived from the Operation of Construction (AM7D): replace AM by A.

[**11**] **Title:** *Aspects of representation theory for finite Lie groups*

Concept analysis: Representation / Theory / Finite / Lie / Groups

Chain: Groups (ASA) — Lie groups (ASJ) — Finite (ANJ) — Representation (AM9S) — Theory (AM3A)

Classmark: ASJ NJ 9S 3A (ASJ NJ9 S3A)

Comments:

1. ASJ implies its containing class ASA — so latter does not appear.
2. Use of Property (Finite) as specifier: drop A.
3. Use of Relation (Representation) in normal role: drop AM.

[**12**] **Title:** *On the characterization of local fields by their absolute Galois groups*

Concept analysis: Local / Fields / Absolute / Galois / Groups / Algebra

Chain: Fields (ASV) — Local (ANF) — (Subsystems) Groups (ASA) — Galois (ANH) — Absolute (AN4Q)

Classmark: ASV NF F SA NH N4Q (ASV NFF SAN HN4 Q)

Comments:

1. Use of Property (Local) as specifier: drop A.
2. Groups (taken from Systems AR/AW) acts as Subsystem: replace the initial A by F.
3. Use of Properties (Galois and Absolute) as specifiers of the subsystem: drop initial A in both cases.

[**13**] **Title:** *Approach to algebraic K-theory*

Concept analysis: Algebra / K-theory / Cohomology / Topology

Chain: Algebras (ATA) — Topological (AVJ) — Cohomology (AMK) — Special theories (AM3B)

Classmark: ATA VJ A K 3B (ATA VJA K3B)

Comments:

1. Use of a system (AVJ) to specify another system.
2. Use of Relation (Cohomology) (AMK) in normal role, replacing initial 'AM' by intercalator 'A'.
3. Use of AM3 B (Special theories) to indicate a theory special to a context; K-theory specifically enumerated under Cohomology (AMK).

[**14**] **Title:** *Computer analysis of rounding errors in matrix algorithms*

Concept analysis: Computers / Roundings / Errors / Matrices / Algorithms

Chain: Matrices (ATB) — Algorithms (AM68) — Errors (AM35) — Rounding (derived from AM84) — (using) Computers (AM2N)

Classmark: ATB 68 35 K4 2N (ATB 683 5K4 2N)

Comments:

1. Use of Forms of mathematical presentation (Algorithms, Errors) in normal role: drop AM in both cases.
2. Example of using unscheduled term ('rounding'). This is an operation, but is acting in the role of specifier of a type of error.
3. To add an operation not at present in the mathematics schedule, AM84 is provided; but since it is acting here as a specifier, AM8 is replaced by the intercalator K.
4. The above way of accommodating a new specifier could be used if many types of error were likely to be written about and the indexer wished to maintain a meticulously classified order — i.e. showing that this class reflects a 'Type by operation'.
5. An alternative, and simpler way of inserting a 'new' (unscheduled) term would be to use X or Y, which are always available for enumeration of special types under any concept. Note that letters A/W must always be held open to allow qualification or specification by Auxiliary Schedule AM1. So to AM3 5 Errors could be added AM3 5X or AM3 5Y for rounding errors. In this case the full classmark would be ATB 68 35X 2N.
6. The point of this example is to demonstrate that BC2 is open to almost endless expansion, retaining a maximum degree of systematic order, or (where such meticulousness seems to be uncalled for) keeping the additional notation as brief as possible.

[**15**] **Title:** *Some partitions of a symmetric matrix*

Concept analysis: Partition / Symmetric / Matrices

Chain: Matrices (ATB) — Symmetric (ANOJ) — Partition (AM7W)

Classmark: ATB NOJ 7W (ATB NOJ 7W)

Comments:

1. Use of Property (Symmetric) as specifier: drop initial A.
2. Use of Operation (Partition) in normal role: drop initial AM.

[**16**] **Title:** *Decomposition of nilpotent Lie algebras*

Concept analysis: Decomposition / Nilpotent / Lie / Algebras

Chain: Algebras (ATA) — Lie algebras (ATC) — Nilpotent (AO5) — Decomposition (AM8GM)

Classmark: ATC O5 8GM (ATC O58 GM)

Comments:

1. Use of Property (Nilpotent) as specifier: drop initial A.
2. Use of Process (Decomposition) in normal role: drop AM.

[**17**] **Title:** *Models of rational surfaces over arbitrary fields*

Concept analysis: Models / Rational / Surfaces / Fields / Algebraic geometry

Chain: Algebraic geometry (ATG) — Surfaces (ATJVS) — Rational (AMY) — (Subsystem) Fields (ASV) — Models (AM3L)

Classmark: ATJ VS MY F SV 3L (ATJ VSM YFS V3L)

Comments:

1. ATJ VS implies its containing class ATG — so lattter does not appear.
2. Use of Property (Rational) as specifier: drop initial A.
3. Use of System (Fields) as subsystem: replace A by intercalator F.
4. Use of Form of mathematical presentation (Models) in normal role: drop AM.
5. The distinction between primary system and secondary system (to be treated as subsystem) sometimes has to be decided on pragmatic grounds. Usually, the more general system (here, field) is best treated as the subsystem, the more narrowly defined class (here, surfaces) representing the focus of attention.

[**18**] **Title:** *R-vector fields on metric manifolds*

Concept analysis: R-vector / Fields / Metric / Manifolds / Global differential geometry

Chain: Differential geometry (AUM) — (Types) (AUV) — Global (ANI) — (Subsystem) Manifolds (AUG) — Metric (AON) — (Subsystem) Fields (ASV) — Vector (AQH) — R-vector (AQHX)

Classmark: AUV NI F UG ON F SV QHX (AUV NIF UGO NFS VQH X)

Comments:

1. AUV implies its containing class AUM — so latter does not appear.
2. Use of Property (Global) as specifier: drop initial A.
3. Use of intercalator F twice, to introduce 2 successive subsystems (Manifolds, Fields); initial A dropped in both cases.
4. Use of Property (Metric) as specifier: drop initial A.
5. Use of Entity (R-vector) as specifier: drop initial A.
6. The letter X is always available for the extension of a concept by a special type; see introduction, section 10.66.

[**19**] **Title:** *Moduli, deformations and classifications of compact, complex manifolds*

Concept analysis: Compact / Complex / Manifolds / Topology

Chain: Topology (AVJ) — (Other subsystems) (AVO) — Manifolds (AUG) — Compact (ANR) — Complex (AMW)

Classmark: AVO G NR MW (AVO GNR MW)

Comments:

1. AVO reflects interruptions in the normal synthesis of topology — see the schedule.
2. A further modification of normal synthesis at AVO allows Manifolds to be indicated simply by G (not FUG as would normally be the case for it as a subsystem, following intercalator F). The normal appearance of F can be seen at AVJ F, before the interruptions began.

3. Use of Properties (Compact, Complex) as specifiers, dropping initial A in each case.
4. The somewhat arbitrary mixture of qualifying concepts (moduli, deformations, classifications) is not intended as a systematically related class. If the library had a special interest in this area, three separate catalogue entries could of course be made, adding one of the three concepts each time to the basic classmark given here.

[**20**] **Title:** *On the characteristic ring of extensions of Lie algebras*

Concept analysis: Ring / Extensions / Lie / Algebras / Differential topology

Chain: Topology (AVJ) — (Types of topologies) (AVP) — Differential (derived from AM7R) — (Subsystem) Algebras (ATA) — Lie algebras (ATC) — Extensions (AM9E) — (Subsystem) Rings (ASM)

Classmark: AVP J R F TC 9E F SM (AVP JRF TC9 EFS M)

Comments:

1. Use of the Operation 'differentiation' as a specifier, replacing AM7 by intercalator J.
2. Use of intercalator F twice, to indicate 2 successive subsystems (Lie algebras, Rings), dropping initial A in both cases.
3. Use of Relation (Extensions) in normal role, dropping AM.
4. Demonstrates complexity of relations between subsystems and components; e.g. a ring of extensions must be distinguished from extensions of a ring.

[**21**] **Title:** *Singularity points of smooth mappings*

Concept analysis: Singular points / Smooth / Mappings / Analysis

Chain: Analysis (AW) — Mappings (AM8K) — Smooth (ANM) — Points (AQB) — Singular points (AQCG)

Classmark: AW 8K NM E CG (AW8 KNM ECG)

Comments:

1. Use of Relation (Mappings) in normal role: drop AM.

2. Use of Property (Smooth) as specifier: drop A.
3. Use of Element (Points) in normal role: replace AQ by E.
4. The document makes it clear that the context is Analysis (AW). Mappings as a completely general concept would go at AM8K.

[**22**] **Title:** *Inequalities for fractional derivatives on the half-line*

Concept analysis: Inequalities / Fractional / Derivatives / Real functions

Chain: Analysis (AW) — Functions (AM8L) — Real (AN3) — Derivatives (APU) — Fractional (ARKI) — Inequalities (AM9N)

Classmark: AW 8 N3 D U RKI 9N (AW8 N3D URK I9N)

Comments:

1. The title leaves as implicit the fact that the context is real functions. This is an example of where the indexer may need the help of other indexing data (from bibliographies, indexing and abstracting services, etc).
2. At AM8 L Functions have a special 'Add . . . ' instruction in order to get briefer classmarks for types of functions. So Real functions is AM8 N3, not AM8 LN3.
3. Use of Element (Derivatives) in normal role: replace AP by D.
4. Use of Systems concept (Fractions) as specifier: drop initial A.
5. Use of Relation (Inequalities) in normal role: drop AM.

[**23**] **Title:** *On the solutions of a class of third-order systems*

Concept analysis: Solutions / Third-order / Ordinary differential equations

Chain: Analysis (AW) — Ordinary differential equations (AWF) — Third order (ANDA) — Solutions (APG)

Classmark: AWF NDA D G (AWF NDA DG)

Comments:

1. Title leaves unstated the precise 'system' considered.
2. Use of Property (Third order) as specifier: drop A.
3. Use of Element (Solutions) in normal role: replace AP by D.
4. AWF reflects an interruption in the normal synthesis in Analysis.

13.4 Further comments on practical classification

13.41 Probably the greatest problem facing the indexer is the difficulty of the subject itself. One main reason for the development of faceted classification for information retrieval was the need librarians have to handle materials on a wide range of topics, in most of which they have no particular competence. By clearly articulating the information structure of each subject into explicit facets and arrays and giving a few clear rules for combining concepts from different facets and arrays a predictable patten is established in each and every subject, and the locating of any item in the information store is greatly facilitated.

13.42 But the highly abstract and specialized nature of mathematics which has made it the most difficult class to design in the whole of BC2 also makes it the most difficult to apply. So the indexer is well advised to get as much guidance as possible from published sources (bibliographies, indexing and abstracting services, etc). The title of a paper or document will not always make clear the main branch or sub-branch providing the context of the problem considered; this is often regarded as more or less implicit (some of the practical examples above demonstrate this — e.g. title [23]). But once the relevant class in the primary Branches facet is established (and most of the literature begins with a concept from the Branches facet) the regular and predictable structure of the schedule will usually take care of the detailed qualification and/or specification called for.

13.5 Care must be taken to recognize when a concept is acting as a specifier and not in its normal facet role. Natural language gives some help here, in that a specifier frequently takes an adjectival form, as in Homological algebra. But this is not always the case and one function of the extensive enumeration of compounds (see Section 10.5) is to give some guidance to the indexer in this matter.

13.6 Specificity in summarization reflects the problem of broad versus close classification (which is not peculiar to mathematics). A specific subject description summarizes the subject of a document as exactly as possible, with every component concept necessary to its definition recognized and at its most precise hierarchical level.

13.61 The completely synthetic notation has an enormous power of highly specific indexing. This problem is considered in the Introduction to BC2 (Section 7.46 Broad and close classification) and a library using BC2 should decide consciously on the policy it wishes to follow. At the book level high specificity is achievable with reasonable brevity (certainly as briefly as in other systems). But at the level of research papers, as the examples above demonstrate, the classmarks tend to become lengthy owing to the number of components making up the compound class which summarizes the exact subject.

13.62 Although this gives the advantage of locating a subject precisely and predictably however large the file grows, some indexers may wish to consider some degree of abbreviation. If some abbreviation is decided on, this is best done on a conceptual basis rather than by some arbitrary choice of the number of characters to be tolerated in a classmark; e.g. title [18] above could be abbreviated to

Differential geometry — Global — Manifolds — Metric (AUV NIF UGO N)
or even to
Differential geometry — Global — Manifolds (AUV NIF UG).

13.7 A special aspect of specificity is the ability of a fully synthetic classification like BC2 to provide classmarks for classes which are potential rather than existent (in the literature) — or, at the very least, not already given as compounds in the schedule. For example, computational matrix algebras does not appear in the schedule, but can be easily synthesized as ATB JH (the 'J' replacing AM7 from AM7 H for computation — the Operation now functioning as a specifier). Or, Power residues of integers could be synthesised at ARK FDQ PN (where ARK FDQ is Integers — Residues, and the PN is taken from the Element APN Powers).

13.8 It may be noted that a long classmark is more easily sought in a classified file if all classmarks are broken up into regular groups of three characters, separated by a space. Although this tends to obscure the exact steps by which the class has been built up (the structural expressiveness of the notation) this feature is not a matter which interests the searcher of a file. The ease of searching when there is a regular pattern of groups of three outweighs the disadvantages. It also helps when marking books or papers for physical storage on shelves or in files.

14 Multiple entry in the classified catalogue or bibliography

14.1 This is described fully in the Introduction to BC2 (Section 7.62) and only the essentials are given here. But the peculiarities of the Mathematics class demand certain modifications — see Sections 14.3/.7 below.

14.11 Books on the shelf are forced to observe one citation order — each book can go in one place only. Everything on a concept in the primary facet is kept together, but at the expense of scattering material on the other facets.

14.12 A catalogue, however, can have several entries for each document and thereby provide several different arrangements. For example, a work on congruence and the ideals of lattices could be given 3 identical entries — but each would file in a different position, reflecting a different citation order:

Lattices — Ideals — Congruence (classmark ARR–ASS–AM9I)
Ideals — Congruence — Lattices (classmark ASS–AM9I–ARR)
Congruence — Lattices — Ideals (classmark AM9I–ARR–ASS)

14.13 Each element in the chain is represented by its full classmark — i.e. second and subsequent elements would not be modified as in the examples in practical classification. Each element is then brought to the front by a rearrangement, as shown above. The procedure is often called 'rotation of entries' using the analogy of figures on a clockface, with each one taken in turn to begin a new combination which nevertheless observes the same relative clockwise order of its figures or elements.

14.2 A practice favoured by some BC2 libraries is to use the normal classmark (reflecting the economies of retroactive notation) for the physical arrangement of the document itself — e.g. putting the classmark on the back of the book. The longer, 'articulated' form shown above is used on the catalogue entries. Each entry carries a clear note: 'Shelved at –' to show the document's shelf location.

14.3 The above procedure, which is quite mechanical once the initial classmark has been assigned, is effective for all subject descriptions in which each element acts in its normal facet role. But when one or more terms acts in a different role (in particular, as a specifier) a purely mechanical 'rotation' can produce unhelpful constructions.

14.31 For example, title 8 in Section 13.3 is Inverse elements in finite symmetric semigroups. Its chain could be stated simply in terms of the classmarks for its components as in (1) below.

This chain could then be permuted to give additional entries; the full set of entries would be:

1.	ARY	—	ANO J	—	ANJ	—	APA	—	AM9 P	
	Semigroups	—	Symmetric	—	Finite	—	Elements	—	Inverse	
2.	ANO J	—	ANJ	—	APA	—	AM9 P	—	ARY	
	Symmetric	—	Finite	—	Elements	—	Inverse	—	Semigroups	
3.	ANJ	—	APA	—	AM9 P	—	ARY	—	ANO J	
	Finite	—	Elements	—	Inverse	—	Semigroups	—	Symmetric	
4.	APA	—	AM9 P	—	ARY	—	ANO J	—	ANJ	
	Elements	—	Inverse	—	Semigroups	—	Symmetric	—	Finite	
5.	AM9 P	—	ARY	—	ANO J	—	ANJ	—	APA	
	Inverse	—	Semigroups	—	Symmetric	—	Finite	—	Elements	

14.32 Clearly, except in case (4), the permuted entries destroy the sense of the index descriptions. This is because they break down the bonds between the two concepts specified (Semigroups and Elements) and their respective specifiers. A good index description should always be 'readable' by the mental addition of ordinary language connectives. In particular, reading the chain backwards should produce a fair English sentence. This is true of the standard citation order governing the preferred order (the first chain). Read backwards, this gives: Inverse elements (in) finite, symmetric semigroups. It is true also of entry (4). This could be read backwards as: Finite, symmetric semigroups (with) inverse elements; or forwards as: Elements (which are) inverse (in) semigroups (which are) symmetric and finite. The other three permuted entries cannot be 'read' in any sensible fashion.

14.33 So mechanical permutation, disregarding the way the various concepts in each entry are linked conceptually, must be modified so that these bonds are recognized. We are assisted here by a number of well established indexing rules (referred to in Section 7.25). These link properties with the thing possessing them, operations with the thing acted on, agents with the operation they assist, specifiers with the thing they specify and so on — see Section 14.5 below.

14.4 Maintaining bonds between linked terms

14.41 If two or more terms are linked in a particular relationship (listed in Section 14.5 below) they should be treated as a unit and kept together throughout the permutation of entries.

14.42 Such units are best indicated (on the cataloguing work-sheet) by enclosing the linked terms within brackets. This is done as an additional step following the making of the chain in citation order — see Section 13.1.

Semigroups (ARY) — Symmetric (ANOJ) — Finite (ANJ) — Elements (APA) — Inverse (AM9P)

Here, Symmetric and Finite clearly specify a type of Semigroup and Inverse specifies a type of Element. These links can be shown as follows:

(Semigroups — Symmetric — Finite) — (Elements — Inverse)
a b c d e

Marking each term with a small letter simplifies the writing out of the different chains.

14.43 Each term is now brought to the front position, but still kept with the other linked terms in its unit:

abcde (Semigroups — Symmetric — Finite — Elements — Inverse)
bacde (Symmetric — Semigroups — Finite — Elements — Inverse)
cabde (Finite — Semigroups — Symmetric — Elements — Inverse)
deabc (Elements — Inverse — Semigroups — Symmetric — Finite)
edabc (Inverse — Elements — Semigroups — Symmetric — Finite)

Note that the first bracketed set is permuted first, then the second one.

14.44 When there are two or more specifiers, (e.g. b and c above) the thing specified always comes second (as above), whichever specifier is brought to the front. The relationship of the specifier to the thing it specifies is closer than its relationship to another specifier.

14.5 Relations between concepts which need their links maintained

Treat as a unit the terms linked by the following relationships:

1. Types of anything and their specifiers (e.g. Symmetric semigroups)
2. Operations and the thing operated on (e.g. Matrices — Partition)
3. Process and the thing displaying them (e.g. Sequences — Distribution)
4. Properties and the thing displaying them (e.g. Coordinates — Density)
5. Elements/Entities and the thing containing them (e.g. Integers — Sequences)
6. Subsystems and the thing containing them (e.g. Ideals — Lattices)

14.51 Note that two facets do not feature directly in the above: Relations, which are defined as multi-directional (see note preceding AM8 J in the schedule); and Mathematical forms and methods, which usually relate to the whole subject of an index description and not any one part. The latter facet is, of course, cited last in normal BC2 practice and many of its terms may not be thought worth bringing to the front for a separate entity — e.g. it is difficult to think of a searcher wanting together everything referred to as theory.

14.6 Even with the links maintained by the above rules, multiple entry will still occasionally result in the confounding of different relationships; e.g. it will not be able to distinguish ring of extensions from extensions of rings.

14.7 Further examples of multiple entry

14.71 All the titles are taken from Section 13.3. For ease of demonstration, full classmarks are replaced by simple a,b,c etc.

14.72 **Title 4:** Classes of graphs that are critical with respect to coverings.

Chain: Graphs $\rightsquigarrow$ a — Critical points $\rightsquigarrow$ b — Coverings $\rightsquigarrow$ c

Permuted entries: abc / bca / cab

Comments: No bonding is required here, so permutation is purely cyclic.

14.73 **Title 3:** Coordinate density of sets of vectors

Chain: Sets — Vectors — Coordinates — Density

Chain with brackets (to show that Vectors specify Sets and Density is a property of Coordinates):
(Sets $\rightsquigarrow$ a — Vectors $\rightsquigarrow$ b) — (Coordinates $\rightsquigarrow$ c — Density $\rightsquigarrow$ d)

Permuted entries: abcd / bacd / cdab / dcab

Comments: Each of the two units is permuted in turn; each term comes to the front in one entry.

14.74 **Title 6:** Connection between congruence relations and the neutral ideals of lattices.

Chain: Lattices — Ideals — Neutral — Congruence
Chain with brackets (to show that Ideals are subsystems of Lattices but at the same time are qualified by the property Neutral):
(Lattices ⇝ a — (Ideals ⇝ b — Neutral ⇝ c)) — Congruence ⇝ d

Permuted entries: abcd / bcad / cbad / dabc

Comments:

1. Demonstrates one bracketed unit (bc) within a larger unit (abc).
2. The brackets round (bc) mean that the two concepts should not be separated in any permutation. If the brackets were only round (abc) the third permuted entry would be (by purely cyclic order) cabd, not cbad, and b and c would be separated.
3. Demonstrates the independence of a Relations facet concept; congruence links both ideals and lattices.

14.75 **Title 18:** R-vector fields on metric manifolds

Chain: Differential geometry — Global — Manifolds — Metric — Fields -R-vector
Chain with brackets (see Comments):
(Diff. geom. ⇝ a — Global ⇝ b) — ((Manifolds ⇝ c — Metric ⇝ d) — (Fields ⇝ e — R-vector ⇝ f))

Permuted entries: abcdef / bacdef / cdefab / dcefab / efcdab / fecdab

Comments:

1. R-vector fields is a subsystem of metric manifolds, which is itself a subsystem of global differential geometry. Consequently, the two subsystems need to be linked, as shown.
2. So the last two entries (in which ef is rotated independently of the other terms) show ef being followed by cd; if normal cyclic order were followed the order would be efabcd and feabcd.

15 Mathematics and Statistics in BC2 compared with BC1

15.1 The reasons for the radical nature of the revision of BC1 are considered in detail in the Introduction to BC2. A comparison of BC2 with BC1 will quickly reveal that the revision in this class has been extremely radical. When comparing any two classification schedules the central criteria are always:

1. what the citation order is and whether it is consistently observed;
2. the degree to which the terms are organized clearly into separate facets and arrays — this being the essential basis for the operation of a consistent citation order.

15.2 Facets and arrays in BC1

15.21 There is little evidence in BC1 of any awareness of the need to recognize these. The absence of any special systematic schedule in mathematics is significant here, since in nearly all those classes of BC1 where categorization is prominent it is reinforced by such an auxiliary schedule, which then allows extensive synthesis.

15.22 Division of mathematics is almost immediately into the traditional branches of mathematics and there is no body of separate classes which give mathematical methods, operations, processes, relations, properties, etc., per se, independently of the branches. It is true AM is called 'General and miscellaneous' and miscellaneous it certainly is. It contains mostly concepts from the Common Subdivisions (e.g. courses, tables of computation, etc., mathematical puzzles and recreations) but includes in its middle Higher mathematics — and this before elementary arithmetic and algebra have appeared.

15.23 Under Arithmetic (AN) occurs Numbers and number systems (ANC/F); but this array is incomplete and most number systems are under Elementary algebra (AO) at AOE/J. The rest of AN is a mixture of operations (ANH/J), elements and entities (ANK/O) and more operations together with relations (ANQ/S). AP contains Elementary algebra and consist wholly of equations. The separation into their elements and entities (polynomials, solutions, roots, etc) and types of equations is reasonably systematic. AQ (Higher algebra) mixes up concepts from the arrays of Relations (e.g. forms, determinants), Elements and Entities (e.g. invariants, tensors, series) and systems structures (e.g. groups, types of algebras). So general a concept as transformation appears only under invariants (as linear transformations AQQ) under Projective geometry (ATM) and as a specifier at AQI Transformation groups — but it has no general place. AR (Analysis) scatters various types of calculi throughout the class and only equations are kept together in a consistent manner.

15.24 An example of the sort of minor inconsistency which occurs throughout the mathematics schedule is the appearance under integral calculus of its main operation integration but the omission under differential calculus of its main operation differentiation. Similarly, in Geometry (AT/AV) we find under analytical geometry curves and surfaces, but under descriptive geometry only surfaces — no curves.

15.25 At AW/AX, separating mathematics from statistics, is Mensuration and Metrology. Trigonometry is found under the former (at AWF/O) — a serious confusion of pure and applied mathematics in a system in which they are kept well apart as a matter of principle. In BC2 metrology will appear in Class U/V Technology.

15.26 In AY Statistics there is no clear separation of statistical models (the fundamental product in statistics and its primary facet) and the particular attributes, processes, operations, etc. which serve them.

15.3 Citation order

15.31 Without a clear organization of terms into facets and arrays a consistent citation order is very difficult to see or to maintain. There is certainly no comprehensive statement of citation order in BC1 — only a number of scattered references or examples which imply a rule in specific instances. For example, the appearance of transformations under Higher algebra and under Projective geometry implies the subordination of a relation to a branch when compounds occur; but although this agrees with the general and explicit rules in BC2 it is nowhere generalized to this effect. Similarly, the concept of roots appears under elementary arithmetic (ANN/O) and under equations in elementary algebra (APH); the implied citation order is Branch — Element, but it is not generalized in a comprehensive rule.

15.32 The note at AVC under Projection, referring to Projective geometry at ATL as a 'distinct subject', acknowledges the notion of a relation acting as a specifier — as does the class AQ1 Transformation groups, already noted. As we have seen (Section 10.43 and elsewhere) this role of specifier is very prominent in mathematics — but BC1 only reflects it in isolated examples, not generalized as a rule of citation order.

15.33 So the only guide to citation order in BC1 is the appearance of enumerated subclasses under different classes. This is in marked contrast to the explicit and comprehensive rules in BC2.

15.4 Filing order

15.41 Without a clear organization of terms into facets and arrays together with a general citation order it is well nigh impossible to judge the consistency or helpfulness of the filing order. All that can be evaluated in BC1 is the order of the branches themselves which constitute the only recognizable facet in the system. Here, the order (Arithmetic… Algebra… Analysis… Geometry) reflects Bliss's emphasis on mathematics as method and the interaction of its two major systems

> Arithmetic and Algebra dealing with numeric and quantitative relations. . . Geometry and Trigonometry, dealing with spatial and metrical relations (Introduction, p77).

He went on to give as the third main branch of mathematics

> Analysis and Theory of Functions, dealing with geometric and algebraic relations combined (ibid).

In view of this, it is curious that BC1 files analysis between Algebra and Geometry; by the principle of inversion (to secure general before special) Analysis should file after both the containing classes to which it conceptually belongs — as it does in BC2.

15.5 Alternative arrangements

15.51 BC1 and BC2 agree in the almost total absence of these in the mathematics schedule. BC1 provides only a few trivial ones (e.g. computation tables in General mathematics (AMT) or Elementary arithmetic (ANT) and one less trivial example (descriptive geometry at ATN, following projective geometry, or at AV, as the last enumerated type of geometry, which Bliss preferred). The one alternative given in BC2 (algebraic geometry, preferred in Algebra but with an alternative under Geometry) is located in Geometry in BC1, but not given as alternative. It should not be forgotten, however, that in BC2 the application of Auxiliary Schedule AM1 to all mathematical concepts, together with its system of distinctive intercalators, in fact allows a high degree of alternative location if the library really wants it.

15.6 Notation

15.61 BC1 makes no provision at all for synthesis in its mathematics class. Only those compounds enumerated in the schedules are provided for and even then they do not always get a distinct classmark; e.g. at AVO (under analytical geometry) is listed a dozen or more types of surfaces, but all get the same classmark.

15.62 The notation in BC2 is, of course, fully synthetic. Any term in any facet or array can be qualified by any other term if desired. The need to allocate notation carefully in order to preserve maximum economy in providing for such compounding has been another major reason for the drastic changes in notation.

15.7 Alphabetical index

15.71 Generally speaking, BC1 does provide an entry under every distinct lead-term. But in addition it duplicates many entries under the entry for their containing class. For example, 'Geometry' has 27 subheadings and nearly all of these appear under their own name — e.g. Algebraic geometry AU. But not quite always — e.g. elementary geometry, logic of geometry and three-dimensional geometry have no entry under their own name.

15.72 Many of the classes enumerated without a distinctive classmark do not appear in the A/Z index — e.g. Group of Galois (API) appears neither under Group nor Galois.

15.73 BC2 uses a basic chain indexing procedure which ensure that every term has its own position as entry word (and adds many compounds also, for reasons explained in Section 11.2).

15.8 Vocabulary in BC1

15.81 Because of the great increase in mathematical knowledge since BC1 was designed (some 50 years ago) the vocabulary is very defective by today's standards. For example, there is not even a distinctive classmark for topology, which appears as one of several un-notated subclasses under ATQ Geometrical configurations; many important algebraic structures (e.g. fields, rings) do not appear at all. BC1 also shows a tendency towards historical and descriptive studies which is out of proportion to the needs of the technical vocabulary.

15.82 If one counts all the concepts actually named in the schedule with or without a distinctive classmark and including the enumerated compounds a rough estimate of some 600 and more terms emerges as the size of the BC1 vocabulary.

15.83 BC2 is a completely synthetic scheme. Consequently, it is very difficult to give a definite size to the vocabulary since the number of classes which can be synthesized is enormous. If we count only the 'elementary' terms (not necessarily single words) and discount all the many compounds which have been enumerated (let alone those which are provided for as potential) as well as the Common subdivisions, we get a rough figure for mathematics (excluding statistics) of some 700/750 terms (over 500 of which are in the preliminary facets AM/AQ and 200 in the Branches facet AR/AW). This modest figure would seem to be quite unrealistic, since the Branches facet is something like four times as big as all the preliminary facets together. The figure excludes such large and heavily divided classes as algebraic number theory, finite groups, Lie groups, differential geometry, differentiable manifolds, functions of a complex variable, harmonic functions, linear operators — all of which are synthesized by terms from the preliminary facets and have therefore not been counted. A rough count of the classes actually enumerated with distinctive classmarks (and counting the compounds as well) is well over 3000.

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